2013 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation \( \frac{dy}{dx} = e^{7(3x^2 - 6x)} \). Let \( y = f(x) \) be the particular solution to the differential equation that passes through \((1, 0)\).

(a) Write an equation for the line tangent to the graph of \( f \) at the point \((1, 0)\). Use the tangent line to approximate \( f(1.2) \).

(b) Find \( y = f(x) \), the particular solution to the differential equation that passes through \((1, 0)\).

STOP

END OF EXAM
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time $t$ days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of $B$. Use $\frac{d^2B}{dt^2}$ to explain why the graph of $B$ cannot resemble the following graph.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$. 
5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function $W$ models the total amount of solid waste stored at the landfill. Planners estimate that $W$ will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25} (W - 300)$ for the next 20 years. $W$ is measured in tons, and $t$ is measured in years from the start of 2010.

(a) Use the line tangent to the graph of $W$ at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of $W$. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25} (W - 300)$ with initial condition $W(0) = 1400$.

END OF EXAM

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6. Solutions to the differential equation \( \frac{dy}{dx} = xy^3 \) also satisfy \( \frac{d^2y}{dx^2} = y^3 \left(1 + 3x^2y^2\right) \). Let \( y = f(x) \) be a particular solution to the differential equation \( \frac{dy}{dx} = xy^3 \) with \( f(1) = 2 \).

(a) Write an equation for the line tangent to the graph of \( y = f(x) \) at \( x = 1 \).

(b) Use the tangent line equation from part (a) to approximate \( f(1.1) \). Given that \( f(x) > 0 \) for \( 1 < x < 1.1 \), is the approximation for \( f(1.1) \) greater than or less than \( f(1.1) \)? Explain your reasoning.

(c) Find the particular solution \( y = f(x) \) with initial condition \( f(1) = 2 \).

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM
5. Consider the differential equation \( \frac{dy}{dx} = \frac{x+1}{y} \).

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for \(-1 < x < 1\), sketch the solution curve that passes through the point \((0, -1)\).

(Note: Use the axes provided in the exam booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the \(xy\)-plane for which \(y \neq 0\). Describe all points in the \(xy\)-plane, \(y \neq 0\), for which \( \frac{dy}{dx} = -1 \).

(c) Find the particular solution \( y = f(x) \) to the given differential equation with the initial condition \( f(0) = -2 \).
5. Consider the differential equation \( \frac{dy}{dx} = \frac{y - 1}{x^2} \), where \( x \neq 0 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

(b) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(2) = 0 \).

(c) For the particular solution \( y = f(x) \) described in part (b), find \( \lim_{x \to 2} f(x) \).

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM
6. Consider the closed curve in the xy-plane given by
   \[ x^2 + 2x + y^4 + 4y = 5. \]
   (a) Show that \[ \frac{dy}{dx} = \frac{-(x + 1)}{2(y^2 + 1)}. \]
   (b) Write an equation for the line tangent to the curve at the point \((-2, 1)\).
   (c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.
   (d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM
5. Consider the differential equation \( \frac{dy}{dx} = \frac{1}{2} x + y - 1 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

(b) Find \( \frac{d^2y}{dx^2} \) in terms of \( x \) and \( y \). Describe the region in the \( xy \)-plane in which all solution curves to the differential equation are concave up.

(c) Let \( y = f(x) \) be a particular solution to the differential equation with the initial condition \( f(0) = 1 \). Does \( f \) have a relative minimum, a relative maximum, or neither at \( x = 0 \)? Justify your answer.

(d) Find the values of the constants \( m \) and \( b \), for which \( y = mx + b \) is a solution to the differential equation.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM
5. Consider the differential equation \( \frac{dy}{dx} = \frac{1 + y}{x} \), where \( x \neq 0 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)

(b) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(-1) = 1 \) and state its domain.

END OF EXAM
5. Consider the differential equation \( \frac{dy}{dx} = (y - 1)^2 \cos(\pi x) \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated. (Note: Use the axes provided in the exam booklet.)

(b) There is a horizontal line with equation \( y = c \) that satisfies this differential equation. Find the value of \( c \).

(c) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(1) = 0 \).