Differentiability

One-Sided Derivatives

$$\lim_{h\to 0^+}\frac{f(x+h)-f(x)}{h}$$

Right-Handed

$$\lim_{h\to 0^-}\frac{f(x+h)-f(x)}{h}$$

Left-Handed

Example

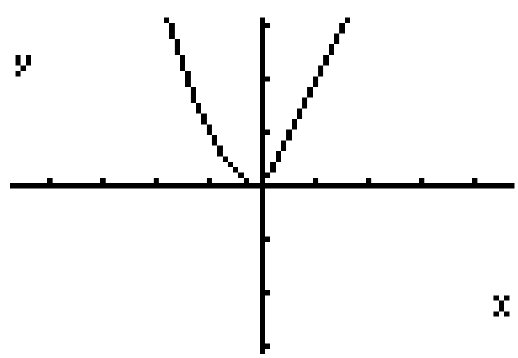
Tell whether the function is differentiable at $x=0$

$$y=\begin{cases} x^2, & x\le 0 \\ 2x, & x>0 \end{cases}$$

$$\lim_{h\to 0^-}\frac{(0+h^2)-0^2}{h}=\lim_{h\to 0^-}\frac{h^2}{h}=0$$

$$\lim_{h\to 0^+}\frac{2(0+h)-2(0)}{h}=\lim_{h\to 0^+}\frac{2h}{h}=2$$

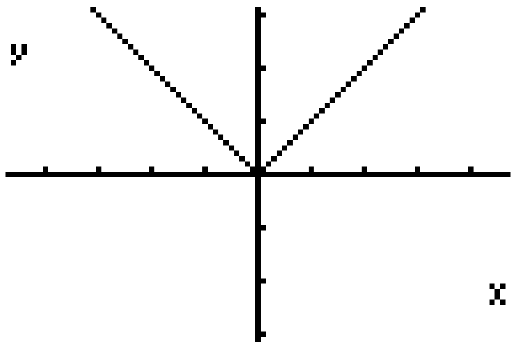
The function does not have a derivative at $x=0$.



-There are several cases where the derivative might fail to exist.

Corner

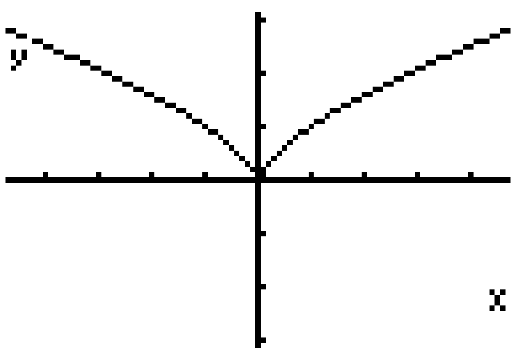
-where the one sided limits differ



$f(x) = |x|$

Cusp

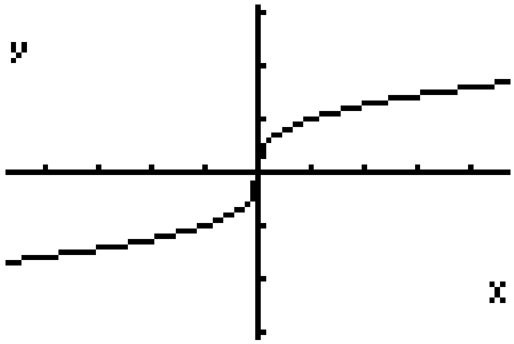
-where the slopes approach ∞ and $-\infty$



$f(x) = x^{2/3}$

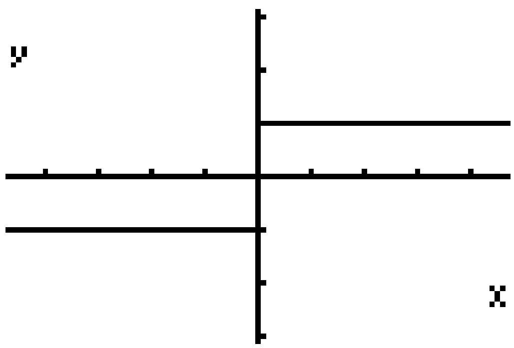
Vertical Tangent

-slopes approach either ∞ or $-\infty$ from both sides



$f(x) = \sqrt[3]{x}$

Discontinuity



Differentiability Implies Continuity

-If f has a derivative at $x = a$, then f is continuous at $x = a$.