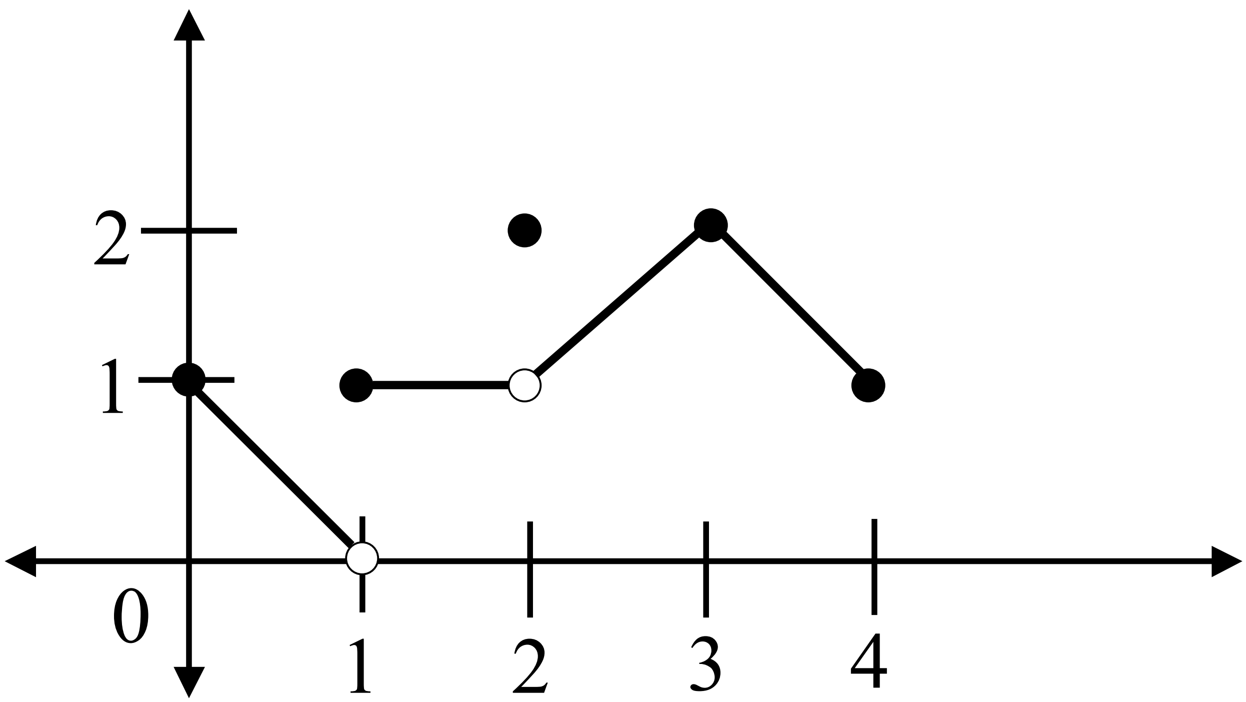
Continuity

-Consider functions that we can draw without lifting our pencil. Then consider "the breaks."

Example-Investigating Continuity



-f is continuous at every point on it's domain $\left[0,4\right]$ except at $x=1$ and $x=2$.

Points that are continuous

At $x=0$ $\lim\limits_{x\to 0^+} f\left(x\right)=f\left(0\right)$

At $x=4$ $\lim\limits_{x\to 4^-} f\left(x\right)=f\left(4\right)$

At $0<c<4, c\neq 1,2$ $\lim\limits_{x\to c} f\left(x\right)=f\left(c\right)$

Points that are discontinuous

At $x = 1$ $\lim_{x \to 1} f(x)$ DNE

At $x = 2$ $\lim_{x \to 2} = 1$, but $1 \neq f(1)$

At $c < 0, c > 4$ these points are not in the domain

Three part Continuity Test

1) $f(c)$ exists

2) $\lim_{x \to 0} f(x)$ is defined

3) $\lim_{x \to 0} f(x) = f(c)$

Example

$$y = \frac{x+1}{x^2 - 4x + 3}$$

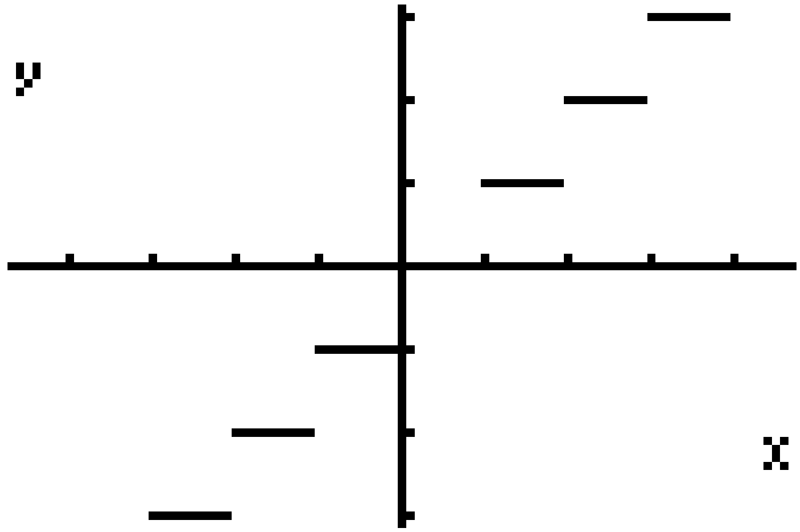
-Think where the problems could be:
- -Divide by 0
- -Negative roots
- -Asymptotes

$c = -1$ $f(c)$ DNE

-Not the end of the story.

-If a function f is not continuous at a point c, we say <u>f is discontinuous</u> at c and c is a <u>point of discontinuity.</u>

Example



-We need to evaluate a 2-sided limit to check for continuity.

$$\lim_{x \to 3^-} \text{int}(x) = 2$$

$$\lim_{x \to 3+} \text{int}(x) = 3$$

Therefore $\lim_{x \to 3} \text{int}(x)$ DNE

-When is $\text{int}(x)$ continuous?

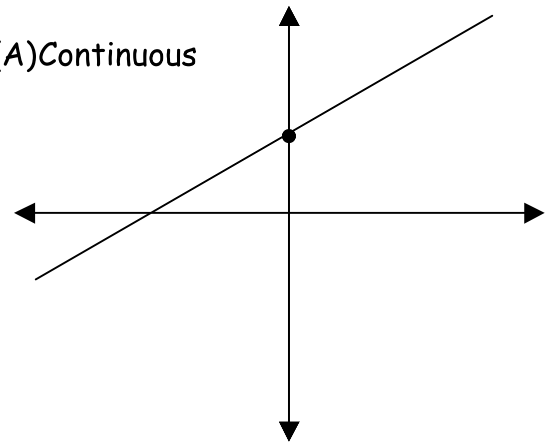
-When is it discontinuous?

Discontinuity Types

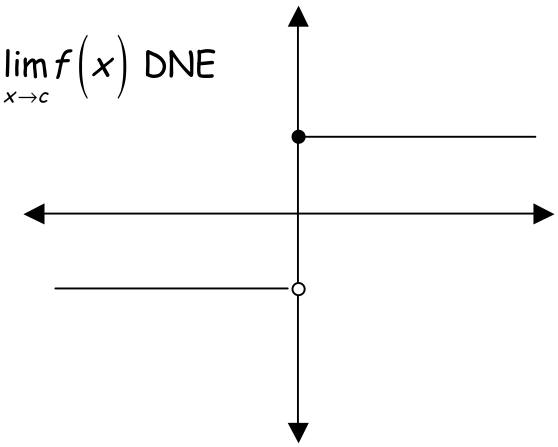
-It's not always just enough to say there is a discontinuity we must sometimes also classify it by type.

-Look at the following graphs and identify what fails to help us identify the type.

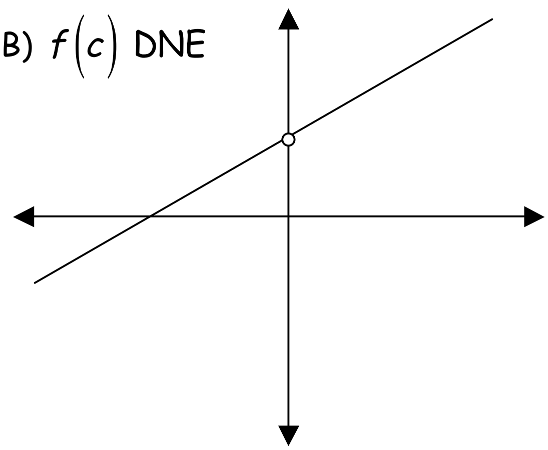
(A)Continuous



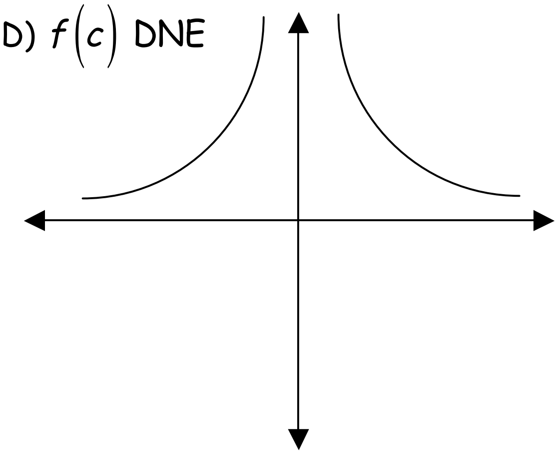
$\lim_{x \to c} f(x)$ DNE



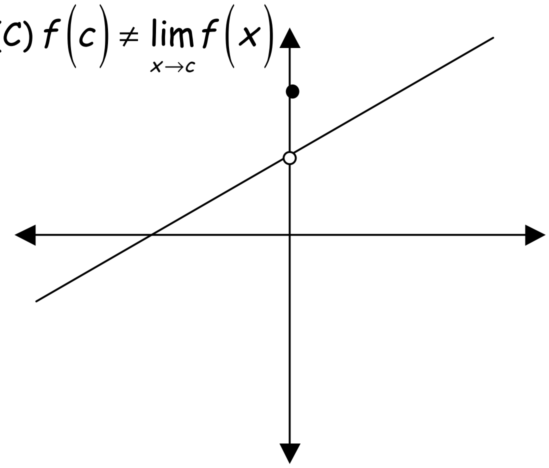
(B) $f(c)$ DNE



(D) $f(c)$ DNE



(C) $f(c) \neq \lim_{x \to c} f(x)$



B and C are <u>removable</u> discontinuities

D is a <u>jump</u> discontinuity

E is an <u>infinite</u> discontinuity

Removing a Discontinuity

Let $f\left(x\right)=\dfrac{x^3-7x-6}{x^2-9}$

1) Factor the denominator. What is the domain of f?

2) Investigate the graph of f around $x=3$ to see that f has a removable discontinuity at $x=3$.

3) How should f be defined at $x=3$ to remove the discontinuity? (TABLE)

4) Show that $\left(x-3\right)$ is a factor of the numerator of f, and remove all common factors. Now compute the limit as x→3 of the reduced form of f.

5) Show that the extended function

$$g\left(x\right)=\begin{cases}\dfrac{x^3-7x-6}{x^2-9}, & x\neq 3\\ \dfrac{10}{3}, & x=3\end{cases}$$

is continuous at $x=3$. The function g is the continuous extension of the original function f to include $x=3$.

Continuous Functions

A function is continuous on an interval iff it is continuous at every point of the interval.

A continuous function is one that is continuous at every point of it's domain.

-Polynomial functions are continuous at very real number c because
$\lim\limits_{x\to c} f\left(x\right)=f\left(c\right)$

-Rational Functions are continuous at every point of their domains. They have points of discontinuity at the zeroes of their denominators.

-The absolute value function is continuous at every real number.

-Exponentials, logarithms, trigonometric, and radical functions are continuous at every point of their domains.

Properties of Continuous Functions

-If the functions f and g are continuous at $x = c$, then the following combinations are continuous at $x = c$

1) Sums $f + g$
2) Differences $f - g$
3) Products $f \bullet g$
4) Constant Multiples $k \bullet f$
5) Quotients $\frac{f}{g}$

Composition of Continuous Functions

If f is continuous at c and g is continuous at $f\left(c\right)$, then the composition $f \circ g$ is continuous at c.

Example

Show that $y = \left|\frac{x \sin x}{x^2 + 2}\right|$ is continuous.

-If we graph $y = \left| \dfrac{x \sin x}{x^2 + 2} \right|$ it looks like it is continuous at every value of x.

-By letting

$$g(x) = |x| \quad \text{and} \quad f(x) = \frac{x \sin x}{x^2 + 2}$$

-We know that the absolute value function g is continuous.

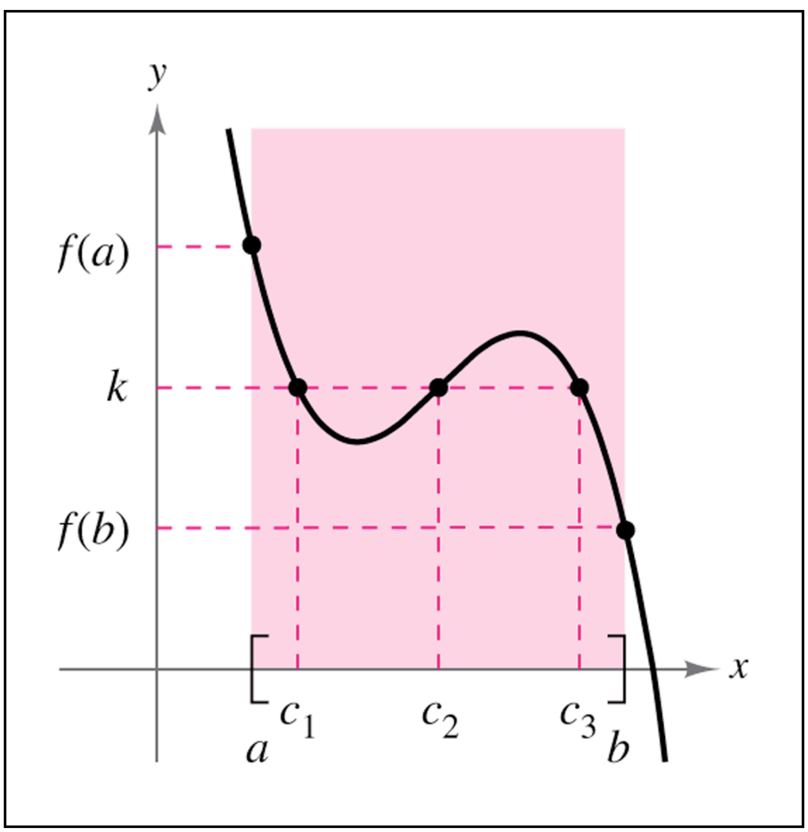
-f is continuous as a quotient

-So the composition is continuous.

Intermediate Value Theorem

-Functions that are continuous on intervals are particularly useful.

-A function is said to have the intermediate value property if it never takes on two values without taking on all the values in between.



-A function $y = f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$. In other words if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a,b]$.

Example

-Is any real number exactly one less then it's cube?

-We know the number must satisfy the equation $x = x^3 - 1$ or equivalently $x^3 - x - 1 = 0$.

-So, we are looking for a zero value on the continuous function $f(x) = x^3 - x - 1$.

-The function changes sign between 1 and 2, so there must be a point c between 1 and 2 where $f(c) = 0$.